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No. 269

THE DISTRIBUTION OF LOADS BETWEEN THE WINGS OF A BIPLANE HAVING DECALAGE

By Richard M. Mock

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Introduction

It is known that in a biplane the load is not distributed equally between the wings. The presence of one wing will affect the lift characteristics of the other wing. A designer must know the total load each wing carries in order that he may design an adequate structure.

The purpose of this thesis is to determine the distribution of loads between the wings of a biplane at various angles of decalage, when the gap/chord ratio is one, and there is no stagger.

Since the distribution of loads between wings is the ratio of the lift of one wing to the lift of the other, the effective lift of each wing will have to be determined. This can be calculated if the effect of the presence of one wing on the lift of the other wing is known. The effective lift of each wing was first investigated, using the vortex theory and later by experiments in the wind tunnel. In order to obliterate a possi-

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ble source of error, two airfoils were used, namely, the U.S.A.27 and the Göttingen 387. Extensive tests were made, using the U.S.A.27 airfoil, and when the results showed a possible error they were checked with the Göttingen 387 airfoil.

The author is indebted to Professor Alexander Klemin and Mr. Frederick Knack for their many helpful suggestions on the theoretical calculations, on the relative values of the vortex theory calculations, and on the wind tunnel results.

The Terms Defined

The decalage, gap, stagger, and angle of attack are measured according to the definitions given by the National Advisory Committee for Aeronautics in their report No. 240 (Reference 1).

The decalage shall be called positive when the lower wing has a smaller angle of attack than the upper wing. The decalage shall be called negative when the lower wing has a larger angle of attack than the upper wing. The angle of decalage is the acute angle between the chords of the wings of a biplane.

The gap is the distance between the planes of the chords of any two adjacent wings, measured along a line perpendicular to the chord of the upper wing at any designated point of its leading edge.

The stagger is the amount of advance of the leading edge of the upper wing of a biplane, triplane, or multiplane, over that of a lower wing, expressed as a percentage of gap or in degree of the angle whose tangent is the percentage just referred to. It is considered positive when the upper wing is forward and is measured from the leading edge of the upper wing along its chord to the point of intersection of the chord with a line drawn perpendicularly to the chord of the upper wing at the leading edge of the lower wing, all lines being drawn in a plane parallel to the plane of symmetry.

All calculations will be made in absolute units employing feet, pounds, and seconds. For a biplane, the following terms shall be used:

C_L = lift coefficient (absolute)

G = gap

b_o = span of upper wing

b_u = span of lower wing

S_o = area of upper wing

S_u = area of lower wing

β = angle of stagger

L = total lift on a wing

D = total drag on a wing

C_{Lo} = effective lift coefficient of upper wing

C_{Lu} = effective lift coefficient of lower wing

If the lower wing of a biplane were removed the upper wing would have a lift coefficient of C_L. When the lower wing is replaced the lift of the upper is affected. The lift coefficient of the upper wing will then be called C_{Lo} or the effective lift coefficient of the upper wing of a biplane. Similarly, the effective lift coefficient of the lower wing will be called C_{Lu}.

The effect of the lower wing on the lift coefficient of the upper will be called ΔC_{Luo}.

$$\Delta C_{Luo} = C_{Lo} - C_L \quad \text{(upper wing alone)}$$

The effect of the upper wing on the lift coefficient of the lower will be called ΔC_{Lou}

$$\Delta C_{Lou} = C_{Lu} - C_L \quad \text{(lower wing alone)}.$$

As the lift coefficient is a function of the lift, all calculations will be made, using coefficients in order that the results may be applicable to other cases.

The Vortex Theory Applied to the Biplane

For purposes of calculation, the airfoil is replaced by a line at one-third of the wing chord. The circulation about the airfoil and the circulation about this imaginary line are equal. The circulation about one wing is disturbed by the presence of the other in two ways. Every wing, about which there is a circulation, has two factors affecting the air around it, the transverse vortex and the tip vortices coming from the ends of the wing. In a biplane these vortices cause a disturbance in the air flow about each wing. The change in the air flow can be attributed to a velocity having a horizontal and a vertical component. The horizontal velocity affects the circulation, while the vertical velocity tends to change the angle of attack.

The velocity at the lower wing, due to the upper transverse vortex, at a point x distance from the center of the wing is (Reference 2a):

$$V_1 = \frac{\Gamma_o \cos\beta}{4\pi G}\left[\frac{\frac{b_o}{2}+x}{\sqrt{\left(\frac{b_o}{2}+x\right)^2+\frac{G^2}{\cos^2\beta}}}+\frac{\frac{b_o}{2}-x}{\sqrt{\left(\frac{b_o}{2}-x\right)^2+\frac{G^2}{\cos^2\beta}}}\right]$$

The velocity at the same point, due to the tip vortices is:

$$V_2 = \frac{\Gamma_o}{4\pi\sqrt{G^2+\left(\frac{b_o}{2}-x\right)^2}}\left[1+\frac{G\sin\beta}{\sqrt{G^2+\left(\frac{b_o}{2}-x\right)^2\cos^2\beta}}\right]$$

The derivations for the above are not necessary in this paper.

It is accepted that the circulation about any wing is:

$$\Gamma_1 = \frac{C_L\ S\ V}{2\ b}.$$

Therefore, the circulation about the upper wing is:

$$\Gamma_o = \frac{C_{Lo}\ S_o\ V_\nu}{2\ b_o}$$

and that about the lower wing is:

$$\Gamma_u = \frac{C_{Lu}\ S_u\ V_\nu}{2\ b_u}$$

where V_ν is the velocity of the air without any external interference.

The change in the horizontal velocity of the lower wing due to the upper wing is ΔV_{ou} (Reference 2b):

$$\frac{\Delta V_{ou}}{V_\upsilon} = - \frac{C_{Lo}\ S_o}{4\ \pi\ b_o\ b_u}\ \mu \quad \text{(Reference 2b)}$$

and similarly,

$$\frac{\Delta V_{uo}}{V_\upsilon} = \frac{C_{Lu}\ S_u}{4\ \pi\ b_o\ b_u}\ \mu \quad \text{(Reference 2b)}$$

The value of μ in terms of the angle of stagger β and λ is given in Figure 1. The value of λ is given below.

The change in the vertical velocity at the lower wing due to the presence of the upper wing is $\Delta\ \alpha_{ou}$ (Reference 2c)

$$\Delta\ \alpha_{ou} = - \frac{C_{Lo}\ S_o}{4\ \pi\ b_o\ b_u} (\upsilon + x) \quad \text{(Reference 2c)}$$

and similarly,

$$\Delta\ \alpha_{uo} = \frac{C_{Lu}\ S_u}{4\ \pi\ b_o\ b_u} (\upsilon - x) \quad \text{(Reference 2c)}$$

The value of υ and x are given in Figures 2 and 3, respectively. The integration of the above equations in order to obtain the values given in Figures 1, 2, and 3, was made graphically by Fuchs and Hopf (References 2d and 2e).

$$\mu = \mu(\lambda_1) - \mu(\lambda_2); \quad \mu(\lambda) = \cos\beta\ (\sqrt{1 + \lambda^2 \cos^2\beta} - 1) \quad \text{(Reference 2f)}$$

$$\upsilon = \upsilon(\lambda_1) - \upsilon(\lambda_2); \quad \upsilon(\lambda) = \sin\beta\ (\sqrt{1 + \lambda^2 \cos^2\beta} - 1) + \\ + \log_e \frac{(1 + \sin\beta)\ (\sqrt{1 + \lambda^2})}{\sin\beta + \sqrt{1 + \lambda^2 \cos^2\beta}}$$

$$x = x(\lambda_1) - x(\lambda_2); \quad x(-\lambda) = \tfrac{1}{2} \log_e (1 + \lambda^2)$$

where $\lambda_1 = \frac{b_o + b_u}{2\ G}$ and $\lambda_2 = \frac{b_o - b_u}{2\ G}$

Since an increase in horizontal wind velocity or in the angle of attack will increase the lift of an airfoil, the increase in lift will be approximately:

$$\Delta L = \frac{\partial L}{\partial V} \Delta V + \frac{\partial L}{\partial \alpha} \Delta \alpha + \underset{\text{negligible}}{D\ \Delta \alpha} \qquad \text{(Reference 2g)}$$

As the lift of a wing is:

$$L = \rho\ C_L \frac{S\ V_\upsilon^2}{2}$$

the increase in lift on the lower wing, due to the presence of the upper wing is:

$$\Delta L = \Delta\ \rho\ C_{Lou} \frac{S_u\ V_\upsilon^2}{2}$$

$$= \frac{\partial\ \left(\rho\ C_{Lou} \frac{S_u\ V_\upsilon^2}{2}\right)}{\partial\ V_\upsilon} \Delta V + \frac{\partial\ (\rho\ C_{Lu}\ S_u\ V_\upsilon^2)}{\partial\ \alpha} \Delta\ \alpha$$

$$\Delta \left(\rho\ C_{Lou} \frac{S_u\ V_\upsilon^2}{2}\right) = \rho\ C_{Lu}\ S_u\ V_\upsilon - \Delta V + \rho \frac{S_u\ V_\upsilon^2}{2} \frac{\partial C_{Lu}}{\partial\ \alpha} \Delta\ \alpha$$

where $\frac{\partial\ C_{Lu}}{\partial\ \alpha}$ is the slope of the lift curve of the airfoil used.

The change in lift of the lower wing, due to the presence of the upper is:

$$\Delta C_{Lou} = 2\ C_L \frac{\Delta\ V_{ou}}{V_\upsilon} \Delta V + \frac{\partial\ C_{Lu}}{\partial\ \alpha} \Delta\ \alpha$$

Substituting, we obtain:

$$\Delta\, C_{Lou} = 2\, C_{Lu}\left[-\frac{C_{Lo}\, S_o}{4\,\pi\, b_o\, b_u}\,\mu\right] - \frac{\partial\, C_{Lu}}{\partial\, \alpha}\left[\frac{57.3\, S_o\,(\nu + x)}{4\,\pi\, b_o\, b_u}\right]$$

(Due to change in horizontal velocity) (Due to change in vertical velocity)

(57.3 changes radians to degrees)

By changing the signs, the effect on the upper wing can be found.

$$\Delta\, C_{Luo} = 2\, C_{Lu}\left[\frac{C_{Lo}\, S_u}{4\,\pi\, b_o\, b_u}\,\mu\right] + \frac{\partial\, C_{Lu}}{\partial\, \alpha}\left[57.3\,\frac{S_u\,(\nu - x)}{4\,\pi\, b_o\, b_u}\right]$$

Application of the Theory

The wings used for the investigation were two metal wind tunnel model 18-inch by 3-inch U.S.A.27 airfoils and two wooden wind tunnel model 18-inch by 3-inch Göttingen 387 airfoils. It has been stated in the introduction that the biplane was investigated at various decalages, when the gap chord ratio was one, and there was no stagger. The angles of decalage that were investigated were: -2°, -1°, $0'$, $+1^\circ$, $+2^\circ$, $+3^\circ$, $+4^\circ$.

The characteristics of each airfoil were determined by testing the airfoil in the wind tunnel. The results of these tests are given in Figure 4. Since in each biplane combination the upper and lower wing have the same span, the same area, and the same lift characteristics.

$$S_u = S_o$$

$$b = b$$

$$\frac{\partial\, C_{Lu}}{\partial\, \alpha} = \frac{\partial\, C_{Lo}}{\partial\, \alpha} = H = \text{slope.}$$

The above can be reduced to the following form (Reference 2h):

$$\Delta\, C_{Lou} = \left(- \frac{\mu\, S}{2\, \pi\, b^2} \right) (C_{Lo}\, C_{Lu}) - \left(\frac{57.3}{4\pi} \right) \left(\frac{\nu + x}{b^2} \right) H\, (C_{Lo})$$

$$\Delta\, C_{Luo} = \left(\frac{\mu\, S}{2\, \pi\, b^2} \right) (C_{Lo}\, C_{Lu}) + \left(\frac{57.3}{4\, \pi} \right) \left(\frac{\nu - x}{b^2} \right) H\, (C_{Lu})$$

In order to know the values of μ, ν and x from Figures 1, 2, and 3, we must determine λ_1 and λ_2.

$$\lambda_1 = \frac{b_o + b_u}{2\, G} = 6$$

$$\lambda_2 = \frac{b_o - b_u}{2\, G} = 0$$

From these values

$$\mu = \mu\, (\lambda_1) - 0 = 5.1$$

$$\nu = \nu\, (\lambda_1) - 0 = 0$$

$$x = x\, (\lambda_1) - 0 = 1.8.$$

$\beta = 0$ as there is no stagger

TABLE I.

U.S.A.27

Theoretical ΔC_{Luo}, effect on upper wing due to lower.

Angle of attack*	-2° decalage	-1° decalage	No decalage	$+1^\circ$ decalage	+2 decalage	$+3^\circ$ decalage	$+4^\circ$ decalage
-4°	-.02281	-.01801	-.01312	--	--	--	--
-2°	-.02147	-.01838	-.01586	-.01251	-.00911	--	--
0°	-.01785	-.0155	-.0137	-.0117	-.01011	-.00798	-.00579
$+2^\circ$	-.0070	-.0064	-.0058	-.0050	-.00445	-.00375	-.00325
$+4^\circ$	+.0077	+.0071	+.0058	+.0058	+.0053	+.0045	+.00417
$+6^\circ$	+.0274	+.0257	+.0240	+.0220	+.0202	+.0183	+.0166
$+8^\circ$	+.0522	+.0499	+.0464	+.0435	+.0402	+.0371	+.0342
$+10^\circ$	+.0810	+.0168	+.0728	+.0695	+.0645	+.0603	+.0560
$+12^\circ$	+.1125	+.1080	+.1035	+.0982	+.0930	+.0889	+.0828
$+14^\circ$	+.0872	+.1345	+.1332	+.1280	+.1225	+.1162	+.1100
$+15^\circ$	--	+.0920	+.1446	+.1404	+.1350	+.1298	+.1230
$+16^\circ$	--	--	+.0279	+.0435	+.0425	+.0408	+.0393

*Angle of attack is measured on upper wing.

TABLE II.

U.S.A.27

Theoretical effective C_{Lo}, abs. lift coefficient of upper wing.							
Angle of attack*	−2° decalage	−1° decalage	No decalage	+1° decalage	+2° decalage	+3° decalage	+4° decalage
− 4°	.17719	.18199	.18688	--	--	--	--
− 2°	.31653	.32962	.33214	.33549	.33889	--	--
0°	.45215	.4545	.4567	.4583	.45989	.46202	.46421
+ 2°	.6024	.6086	.6092	.610	.61055	.611225	.61175
+ 4°	.7557	.7551	.7538	.7538	.7533	.7525	.75217
+ 6°	.9104	.9087	.907	.905	.9032	.9013	.8996
+ 8°	1.0722	1.0699	1.0664	1.0635	1.0602	1.0571	1.0542
+10°	1.231	1.2268	1.2228	1.2195	1.2145	1.2103	1.206
+12°	1.3955	1.391	1.3865	1.3812	1.3760	1.3719	1.3658
+14°	1.4772	1.5245	1.5232	1.518	1.5125	1.5062	1.500
+15°	--	1.523	1.5256	1.5714	1.5660	1.5608	1.5540
+16°	--	--	.9379	.9535	.9525	.9508	.9493

*Angle of attack is measured on upper wing.

TABLE III.

U.S.A. 27

Theoretical ΔC_{Lou}, effect on lower wing due to upper							
Angle of attack*	-2° decalage	-1° decalage	No decalage	$+1^{\circ}$ decalage	$+2^{\circ}$ decalage	$+3^{\circ}$ decalage	$+4^{\circ}$ decalage
-4°	-.02798	-.02601	-.02396	--	--	--	--
-2°	-.05438	-.05127	-.04864	-.04522	-.04168	--	--
0°	-.08275	-.0773	-.0735	-.0693	-.06574	-.0611	-.05635
$+2^{\circ}$	-.1193	-.1136	-.1082	-.1012	-.09615	-.09065	-.0860
$+4^{\circ}$	-.1588	-.1520	-.1450	-.1381	-.1316	-.1230	-.1170
$+6^{\circ}$	-.2038	-.1957	-.1875	-.1794	-.1713	-.1631	-.1554
$+8^{\circ}$	-.2533	-.2460	-.2356	-.2263	-.2166	-.2073	-.1981
$+10^{\circ}$	-.3065	-.2961	-.2858	-.2775	-.2656	-.2550	-.2443
$+12^{\circ}$	-.3603	-.3510	-.3415	-.330	-.3187	-.3094	-.2964
$+14^{\circ}$	-.3003	-.3961	-.3908	-.3808	-.3702	-.3578	-.3453
$+15^{\circ}$	--	-.3090	-.4100	-.4020	-.3917	-.3815	-.3685
$+16^{\circ}$	--	--	-.1965	-.2606	-.2556	-.2491	-.2426

*Angle of attack is measured on upper wing.

TABLE IV.

U.S.A.27

Theoretical effective C_{Lu}, abs. lift coefficient of lower wing							
Angle of attack*	-2° decalage	-1° decalage	No decalage	$+1^\circ$ decalage	$+2^\circ$ decalage	$+3^\circ$ decalage	$+4^\circ$ decalage
-4°	.32002	.24899	.17604	--	--	--	--
-2°	.41562	.35173	.29936	.23978	.15832	--	--
0°	.53225	.4527	.3965	.3337	.28226	.2139	1.4365
$+2^\circ$	.6282	.5664	.5068	.4288	.37385	.31235	.2620
$+4^\circ$	.7242	.6640	.6030	.5419	.4834	.4070	.3530
$+6^\circ$	.8162	.7563	.6955	.6360	.5767	.5169	.4596
$+8^\circ$	.8967	.8490	.7844	.7257	.6664	.6082	.5499
$+10^\circ$	.9765	.9209	.8642	.8175	.7544	.6970	.6387
$+12^\circ$	1.0297	.9860	.9415	.887	.8317	.7856	.7236
$+14^\circ$	.6097	1.0349	.9992	.9562	.9127	.8592	.8047
$+15^\circ$	--	.6010	1.020	.9880	.9453	.9015	.8485
$+16^\circ$	--	--	.7135	1.1704	1.1344	1.0879	1.0404

*Angle of attack is measured on upper wing.

From Figure 4:

$$\frac{\partial\ C_L}{\partial\ \alpha} = .0677 \quad \text{for the U.S.A.27 airfoil,}$$

$$\frac{\partial\ C_L}{\partial\ \alpha} = .0747 \quad \text{for the Göttingen 387 airfoil.}$$

As the airfoils are 18-inch by 3-inch, and the gap chord ratio one

$$S_o = S_u = .375 \text{ sq.ft.}$$

$$b_o = b = 1.5 \text{ ft.}$$

$$G = .25 \text{ ft.}$$

The above equations can be simplified still further to fit this special case.

U.S.A.27 airfoils:

$$\Delta\ C_{Lou} = -\frac{(5.1)\ (.375)}{2\ \pi\ (1.5)^2}\ C_{Lo}\ C_{Lu} - \frac{57.3}{4\ \pi}(1.8)\frac{(.375)}{(1.5)^2} C_{Lo}(.0677)$$

$$= -.1355\ C_{Lo}\ C_{Lu} - .0927\ C_{Lo}$$

$$\Delta\ C_{Luo} = +.1355\ C_{Lo}\ C_{Lu} - .0927\ C_{Lu}$$

Gottingen 387 airfoils:

$$\Delta\ C_{Lou} = -\frac{(5.1)(.375)}{2\ \pi\ (1.5)^2}\ C_{Lo}\ C_{Lu} - \frac{57.3}{4\ \pi}\ \frac{(1.8)(.375)}{(1.5)^2} C_{Lo}(.0747)$$

$$= -.1355\ C_{Lo}\ C_{Lu} - .1023\ C_{Lo}$$

$$\Delta\ C_{Luo} = +.1355\ C_L\ C_{Lu} - .1023\ C_{Lu}$$

The effect of the presence of the lower wing on the lift of the upper, and the effect of the presence of the upper wing on the lift of the lower were calculated for decalage of -2°, -1°, $+1^\circ$, $+2^\circ$, $+3^\circ$, $+4^\circ$ for the U.S.A.27 airfoil and $+2^\circ$ for the Göttingen 387 airfoil. Each biplane combination was calculated for 16 angles of attack. These calculations were made by substituting the values obtained from Figure 4 in the above equations. The angle of attack was measured on the upper wing, thus an increase in decalage causes a decrease in the angle of the lower wing. The numerical results for the calculations for the U.S.A.27 airfoil are given in Tables I and III. The results for the Göttingen 387 airfoil are given below. Figures 5 and 6 show the same values plotted against the lift of the upper wing alone. That is, if the upper wing had the same angle of attack and its lift was not disturbed by the presence of the lower wing.

From the values given in Figure 4, the effective lift of the upper and lower wings were calculated by means of the above results. The numerical results of these calculations for the U.S.A.27 airfoil are given in Tables II and IV, while the results for the Göttingen 387 airfoil are given below:

Angle of Attack	C_{Luo}	C_{Lo} (effective)
-4°		
-2°	-.01115	.38585
0°	-.01155	.5119
-2°	-.0050	.6830
-4°	-.0105	.8785
-6°	-.0330	1.0610

Angle of Attack	C_{Luo}	C_{Lo} (effective)
-8°	$-.058$	1.2290
-10°	$-.0856$	1.3836
-12°	$-.1180$	1.5430
-14°	$-.1413$	1.6283
-15°	$-.1490$	1.6590
-16°	$-.1538$	1.6758

The distribution of loads between wings is the ratio of the effective lift of the upper wing to the effective lift of the lower wing. The distribution of the load between the wings was calculated from the above results. The ratios are plotted in Figure 7.

The Results of the Theoretical Investigation

In discussing the results of the investigation from the view of the vortex theory at this point the wind tunnel test results or any conclusions drawn from them will be omitted.

The variation of ΔC_{Luo} with the C_L of the upper wing alone, as given in Figure 5, show that at angles of attack below 3° for the U.S.A.27 airfoil and below 3.25° for the Göttingen 387 airfoil, the effect of the vertical velocity at the upper wing, due to the lower wing, is greater than the effect due to the horizontal velocity. Above these values the effect due to the horizontal velocity component is greater. The vertical velocity tends to decrease the lift of the upper wing, while the horizontal velocity tends to increase the lift. Therefore, C_{Luo} will be negative below these values and positive above. As the horizontal velocity component is a function of the square of the lift on the upper wing, and the vertical velocity component is directly proportional to the lift on the upper wing, there is a point at which the two components are equal. This is the point when the U.S.A.27 airfoil is at 3° and the Göttingen 387 is at 3.25°. At angles above 3° for the U.S.A.27, and above 3.25° for the Göttingen 387, the effect of the horizontal component is predominant, therefore ΔC_{Luo} will increase with an increase in decalage. Below these angles the effect will be reversed. Therefore, at small angles of attack an increase in decalage increases C_{Luo}.

As both the horizontal and vertical velocity at the lower wing, due to the upper wing, tend to decrease the lift of the lower wing, C_{Luo} will be negative. As an increase in decalage causes a decrease in lift on the lower wing ΔC_{Luo} will have less effect with increased decalage. This phenomenon is shown graphically in Figure 6.

With positive decalage the lift on the upper wing will be greater than that on the lower wing, at all angles of attack of the biplane. This is caused by the upper wing having a larger angle of attack. Neglecting the effect of one wing on the lift of the other, the ratio of the lift of the upper to the lift of the lower will be greater than one. Similarly this ratio will be less than one when there is a negative decalage.

From Figures 5 and 6 it can be seen that at large angles of attack the lift on the upper wing is increased and the lift on the lower wing decreased. Therefore, the ratio of the effective lift on the upper wing to that on the lower wing will be greater than it would be if this effect were neglected. At small angles the decrease in the lift on the lower wing is so much more than the decrease in the lift on the upper wing that the same result is produced, thereby increasing the ratio at all points. At some angles the ratio is not increased as much as at high angles of attack; therefore, the curves in Figure 7 have a general upward slope.

The Experimental Investigation

The Apparatus

In tests in the wind tunnel the most probable source of error is in setting the model to be tested. In this experiment every possible precaution was taken to eliminate any error from this source.

One wing was set in the chuck of the wind tunnel in the usual manner. This chuck rests on the balances. Another chuck was screwed to roof of the wind tunnel directly above the chuck used for the wing just mentioned. The upper chuck was centered accurately by means of a plumb bob. In this upper chuck was set a spindle, offset by a link at 7 inches above the wing. The interfering or dummy wing was mounted on this spindle. The link had a slot in one end to permit a certain amount of freedom in putting the wing in place. It is this interfering or dummy wing that affects the lift on the wing in the lower chuck. By rotating the upper spindle in the chuck the interfering or dummy wing was used either as the upper or lower wing of the biplane. Of course in one position the wing was turned to keep the leading edge into the wind. Figure 8a is a photograph of the apparatus in the wind tunnel.

In the photograph the wings are in position to give readings for the effective lift on the lower wing. Figure 8 gives the dimensions for the apparatus. In the tests a rod was screwed into the lower part of the interfering wing, directly below the spindle. As the rod was long enough to reach the floor of the tunnel, it prevented any possible vibration of the interfering wing. This rod is not shown in the photograph.

Procedure

The investigation was made in 4-foot wind tunnel at New York University. All tests were made with a wind velocity of 40 miles per hour.

The decalage was measured with a pair of drawing dividers having very sharp points. Two fine crosses were scratched on the end of each wing. For each decalage to be investigated a full-scale drawing, showing these crosses, was made of the wings. The wings were placed in the tunnel at approximately the desired decalage. The drawing dividers were then set by placing them on the full scale drawing. With the dividers held over the crosses, scratched on the wings, the wings were moved delicately until the crosses, and therefore the wings, were in the desired position.

It has been stated that the lift was investigated at various angles of decalage, when the gap/chord ratio was one, and there was no stagger. The gap/chord ratio was always equal to one since the full drawing, used to place the wings, was made with that gap/chord ratio.

It may appear that there was a possible source of error in measuring the stagger. According to the National Advisory Committee for Aeronautics definition, there is no stagger when the leading edge of the lower wing lies in a perpendicular to the chord of the upper wing drawn at the leading edge of the upper wing.

In order to prevent the wings from having any stagger, a small jig was used. It consisted of a small "T" made of aluminum. Great pains were taken in making the "T" perfectly square. A spring clamp was screwed to the stem of the "T". This clamp held the stem against the lower side of the upper wing so that one edge of the stem coincided with the chord of the upper wing. One side of the head of the "T" was pressed against the leading edge of the upper wing so that when the leading edge of the lower wing was brought up flush with the other side of the head of the

"T," there was no possibility of any stagger.

One airfoil was set up in the lower chuck (connected to the balance). It was then tested for lift at the ordinary angles of attack.

To measure the effective lift of the upper wing, the wing in the chuck, just described, was used as the upper wing. The lower wing was then put in place. After the "T" shaped jig was clamped to the upper wing, the lower wing was carefully set at the correct decalage by means of the dividers. The lower wing was then locked in place and the decalage checked. After the "T" clamp was removed, the tunnel was started and a reading taken. With the decalage set, the wings were both rotated about the same axis, so that readings could be taken at all angles of attack, without further adjustment. To prevent any error, the decalage was checked before and after the reading at each angle of attack. Great care was taken to keep the wings parallel for each test.

In a similar way the effective lift of the lower wing was measured. The interfering wing was then removed and monoplane readings were again taken on the wing in the lower chuck. The tests were run without removing the wing in the lower chuck between runs. In this way, another possible source of error was removed.

TABLE V.

U.S.A.27

Wind tunnel results for ΔC_{Luo}, effect on upper wing due to lower

Angle of attack*	-2° decalage	-1° decalage	No decalage	$+1^\circ$ decalage	$+2^\circ$ decalage	$+3^\circ$ decalage	$+4^\circ$ decalage
-4°					+.0123		
-2°					-.043		
0°	-.089	-.061	-.044	-.0315	-.017	+.0135	+.059
$+2^\circ$					-.031		
$+4^\circ$	-.086	-.084	-.077	-.068	-.051	-.030	-.022
$+6^\circ$					-.073		
$+8^\circ$	-.128	-.113	-.098	-.091	-.085	-.078	-.073
$+10^\circ$					-.099		
$+12^\circ$	-.158	-.147	-.124	-.113	-.105	-.098	-.083
$+14^\circ$					-.107		
$+15^\circ$							
$+16^\circ$							

*Angle of attack is measured on upper wing.

TABLE VI.

U.S.A.27

Wind tunnel results for effective C_{L_0}, abs. lift coef. of upper

Angle of attack*	-2° decalage	-1° decalage	No decalage	$+1^\circ$ decalage	$+2^\circ$ decalage	$+3^\circ$ decalage	$+4^\circ$ decalage
-4°					.1753		
-2°					.305		
0°	.334	.362	.379	.3915	.406	.4365	.~~472~~ 532
$+2^\circ$					.587		
$+4^\circ$	.662	.664	.671	.688	.697	.718	.726
$+6^\circ$					.810		
$+8^\circ$	.892	.907	.922	.929	.935	.942	.947
$+10^\circ$					1.051		
$+12^\circ$	1.125	1.136	1.159	1.17	1.178	1.185	1.20
$+14^\circ$					1.283		
$+15^\circ$							
$+16^\circ$							

*Angle of attack is measured on upper wing.

TABLE VII.

U.S.A. 27

Wind tunnel results for ΔC_{Lou}, effect on lower wing due to upper

Angle of attack*	-2° decalage	-1° decalage	No decalage	$+1^\circ$ decalage	$+2^\circ$ decalage	$+3^\circ$ decalage	$+4^\circ$ decalage
-4°			+.0055				
-2°					-.0426		
0°	-.043	-.036	-.0335	-.0515	-.1145	-.1157	-.1308
$+2^\circ$					-.099		-.168
$+4^\circ$	-.104	-.105	-.108	-.120	-.1455	-.099	-.107
$+6^\circ$					-.118		
$+8^\circ$	-.177	-.209	-.197	-.211	-.214	-.2045	-.180
$+10^\circ$					-.197		
$+12^\circ$	-.230	-.243	-.246	-.222	-.262	-.252	-.248
$+14^\circ$			-.250		-.232		
$+15^\circ$						-.215	-.237
$+16^\circ$					-.218		

*Angle of attack is measured on upper wing.

TABLE VIII.

U.S.A.27

Wind tunnel results for effective C_{Lu}, abs. lift coef. of lower wing

Angle of attack*	−2° decalage	−1° decalage	No decalage	+1° decalage	+2° decalage	+3° decalage	+4° decalage
− 4			.1685				
− 2					.1204		
0	.572	.494	.4065	.3345	.2335	.1403	1.0322
+ 2							
+ 4	.779	.711	.643	.563	.4725	.422	.180
+ 6							
+ 8	.973	.876	.873	.741	.669	.6115	.316
+10							
+12	.116	1.094	1.037	.995	.888	.833	.568
+14			1.14				
+15						1.068	1.046
+16							

*Angle of attack is measured on upper wing.

The Results of the Wind Tunnel Experiments

The results of the tests with the wing alone are given in Figure 4. It was upon these results that the theoretical calculations were based.

From readings of the lift on the wing, alone in the tunnel, and the lift when the other wing is present, the effect of the presence of one wing upon the lift of the other was calculated. The experimental results for the effective lift of both upper and lower wings are given in Tables VI and VIII, respectively. The effect of the presence of one wing upon the lift of the other, as obtained in the wind tunnel, is given in Tables V and VII. The same results are shown graphically in Figures 9 and 10.

Figure 11 gives the ratio of the effective lift of the upper wing to that of the lower as obtained from these tests (See Fig. 13 for comparison with Fig. 7).

Discussion of the Experimental Results

It can readily be seen that there is a difference between the results obtained by the experimental and by the theoretical investigations. After the nature of the wind tunnel results have been discussed, the reasons for this difference will be explained.

The results obtained in the wind tunnel for ΔC_{Luo}, as shown in Figure 9, are hardly similar to the theoretical results shown in Figure 5. The wind tunnel results for the U.S.A.27 airfoil were calculated, plotted, and found to give some very consistent results. It was thought that there might have been a possible error, since the curves obtained from the wind tunnel investigation did not conform with those obtained by the application of the vortex theory. For this reason a new set-up was made in the wind tunnel, using two Göttingen 387 airfoils as a check on the results obtained by the U.S.A.27 airfoils. The Göttingen airfoils were tested at a decalage of +2°. The results of this test gives a curve very similar to that obtained at +2° decalage with the U.S.A.27 airfoil. Of course these curves do not coincide since the two airfoils have different lift characteristics.

Figure 9 shows that the lower wing reduces the lift of the upper when the angle of attack of the biplane is increased. The lift on the upper wing is increased with an increase in decalage.

The plotting of the wind tunnel results for ΔC_{Luo}, the effect of the upper wing upon the lift characteristics of the lower, give a curve with a slope very similar to that obtained by the theoretical investigation. Though the effect increases with the angle of attack, the lift on the lower wing decreases with an increase in decalage, contrary to the results given by the application of the vortex theory.

Since the lift on the upper wing is decreased at a large angle of attack, the ratio of the lift on the upper wing to that on the lower wing will be less than that obtained theoretically.

As the reduction in lift on the lower wing increases faster than that on the upper wing, the curves will still have the same upward slope as in the theoretical results.

A Comparison of the Results Obtained by the Theoretical and Experimental Investigations

It has been shown that there is a difference between the wind tunnel test results and the calculated results based on the vortex theory.

The fact that the wind tunnel tests were accurate can be proven in two ways. The results obtained from the readings in the wind tunnel, when plotted, gave smooth consistent curves. Secondly, the results were checked by using a different set of airfoils and a new set-up still obtaining the same results. It may be stated here that the possibility of an error in setting up the apparatus is negligible as the wing on which the readings were made was not moved in the chuck between any of the tests, including the tests with the wing alone in the tunnel.

The reason for the difference in the results is in the method of applying the vortex theory. The airfoils of the biplane were replaced by lines at one-third the wing chord. All the calculations were based on the circulation about these lines. When the decalage is varied from -2° to $+4^{\circ}$, and the gap kept constant, these lines come approximately 3/32 of an inch closer together, while the trailing edges of the wing are moved approximately 7/16 of an inch. (The distances between the leading edges remain constant.) The vortex theory as developed by Dr. Fuchs and Dr. Hopf (Reference 2i) does not recognize the movement of these lines or the fact that the distance between the trailing edges is not the same as the gap. The theory has been developed only including the interference between the circulations and the vortices. The Venturi effect produced by having the trailing edges of the wings closer together when there is positive decalage and by having the trailing edges farther apart when there is negative decalage has been neglected. With positive, this Venturi effect tends to increase the velocity of the air between the wings, reducing the circulation about the upper wing and increasing the circulation about the lower wing. It has been shown that the lift of a wing is a function of the circulation. Figures 5 and 6 show an increased lift on the upper wing and a decreased lift on the lower wing, due to the circulation about a line replacing the airfoil and neglecting the Venturi effect. The increased lift is small compared to the decreased lift of the lower wing. (The scale of Figure 6 is five times that of Figure 5.) When the Venturi effect is taken into account, the lift on the upper wing is decreased until it is below the value for the wing alone, making ΔC_{Luo} negative, as in Figure 9. The increased lift of the lower wing will decrease the slope in Figure 6. As the scale of Figure 6 is larger, the effect is not noticeable at first. This is shown in Figure

10. Thus it can be shown that the Venturi effect does affect the lift of the wings. It is well to keep in mind that the effect of the interfering circulations is greater than the Venturi effect.

When the decâlage is increased the Venturi effect increases, increasing the lift of the lower wing, or reducing the effect of the upper wing on the lower wing. This explains the reversed order of the lines in Figure 10 when compared with Figure 6 (See Figure 12).

The reduced value of the ratio of the effective lift of the upper wing to that of the lower wing has already been discussed.

The experimental results bring out many other points in the vortex theory as applied to biplanes. When the wind tunnel results are applied to the equations developed from the vortex theory for ΔC_{Luo}, they show that actually the vertical velocity increases faster than the horizontal velocity. The horizontal velocity tends to increase the lift of the upper wing, while vertical velocity tends to reduce the lift. According to the constants determined by the graphic integration of Dr. Fuchs and Dr. Hopf (Reference 2j), the vertical velocity does not increase as fast as the horizontal velocity. It may be that these constants are not applicable.

In applying the vortex theory, Dr. Fuchs and Dr. Hopf have neglected the fact that when a biplane with no stagger is at a high angle of attack, the same effect is produced as if there were stagger, since one wing meets the wind before the other. This may cause the lower wing to be less affected by the tip vortices of the upper wing, the transverse vortices remaining the same. It may be recalled that the tip vortices tend to reduce the lift of either wing.

As the wing approaches an elliptical loading the vortices are leaving the wing in an increasing amount from the center to the tip of the wing. Dr. Fuchs and Dr. Hopf (Reference 2j) should have made their integration using an elliptical loading when they obtained the constants for Figures 1, 2 and 3.

Conclusions

In a biplane the lift of the upper wing will be greater than that of the lower wing, due to the circulation of the lower wing, increasing the wind velocity at the upper wing and the circulation of the upper wing, decreasing the wind velocity of the lower wing. The increased velocity between the wings due to the Venturi effect tends to decrease the circulation of the upper wing and increase the circulation of the lower wing. The Venturi effect is not as great as that produced by the interference of the circulations.

The tip vortices of each wing tend to decrease the lift of the other. It has been shown that the lift of each wing is decreased, due to the presence of the other, the lift of the lower wing being decreased more than that of the other. Therefore, the total lift of a biplane is less than that of two similar monoplane wings.

Since the lift of each wing is decreased and that of the lower wing is decreased more than that of the upper, the ratio of the effective lift of the upper wing to that of the lower wing will be greater than one except at small angles of attack and when there is no or negative decalage. When the decalage is negative, the lower wing has a greater angle of attack and a greater lift, consequently the ratio is less than one.

The equation for the application of the vortex theory to a biplane should be corrected for the Venturi effect (by replacing the airfoil by more than one line), the effect of the vortices leaving the wing before they reach the end (using elliptical loading), and the effect of the stagger at high angles of attack.

R e f e r e n c e s

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2. Fuchs, Dr. Richard and Hopf, Dr. Ludwig :	(a) Aerodynamik, Section 5, Chapter IV, Kapitel, Der Mehrdecker. Richard Carl Schmidt & Co., Berlin, 1922. Fuchs and Hopf in turn refer to: Betz, A. - Die gegenseitige Beeinflussung zweier Tragflachen, "Zeitschrift für Flugtechnik und Motorluftschiffahrt," R. Oldenbourg, Munchen and Berlin, 1914, S.253, and "Technische Berichte der Flugzeugmeisterei," R. C. Schmidt & Co., Berlin, 14, S.103. Fuchs, R. - "Technische Berichte der Flugzeugmeisterei," loc.cit., II 2, S.177.
	(b) Aerodynamik, loc. cit., p.137 (37) & (37a).
	(c) " " " p.137 (38) & (38a)
	(d) " " " p.143 (Figs. 1 & 2)
	(e) " " " p.144 (Fig. 3)
	(f) " " " p.137 (39), (40) & (41)
	(g) " " " p.144
	(h) " " " p.145 (42)
	(i) " " "
	(j) " " "

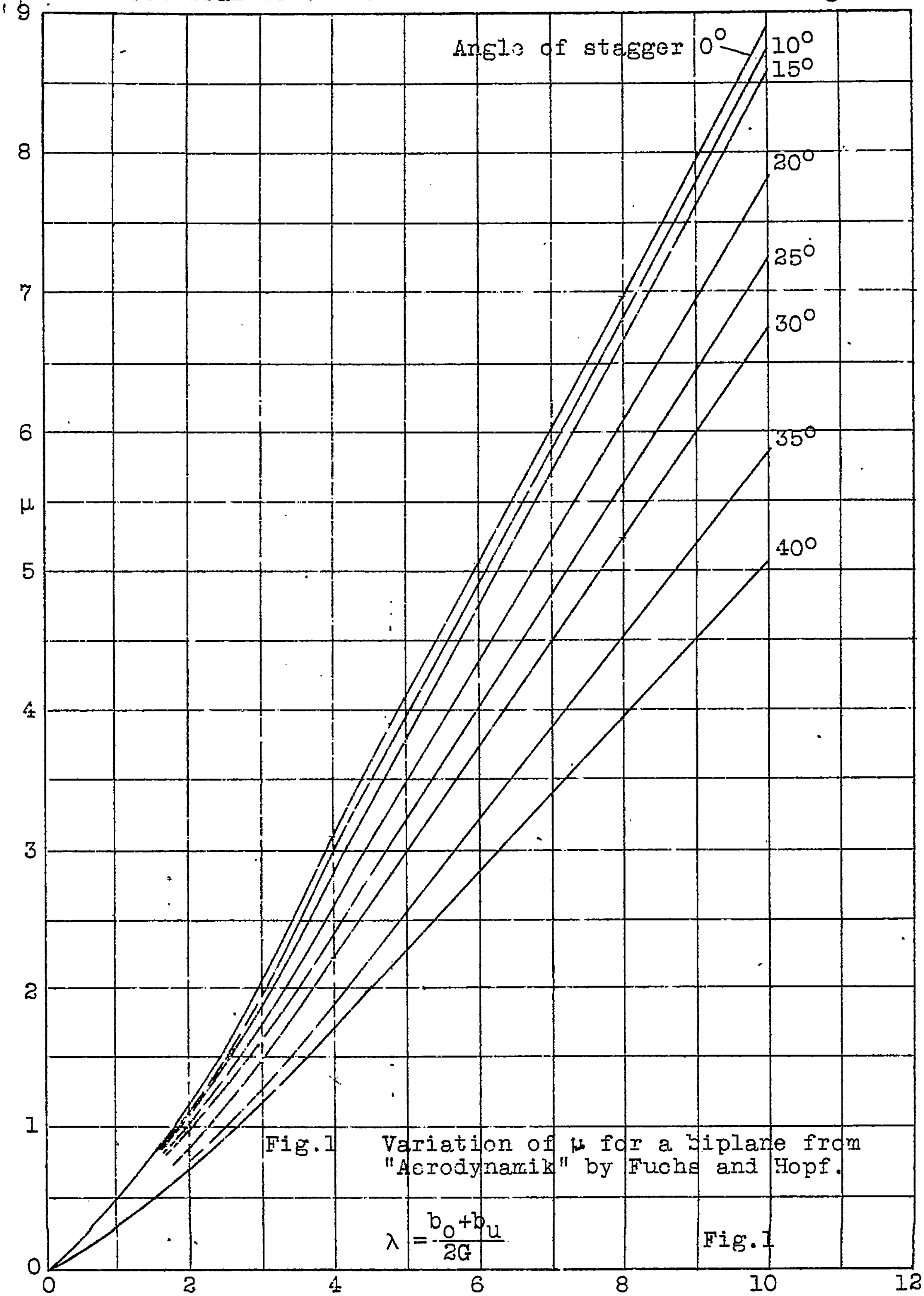


Fig.1 Variation of μ for a biplane from "Aerodynamik" by Fuchs and Hopf.

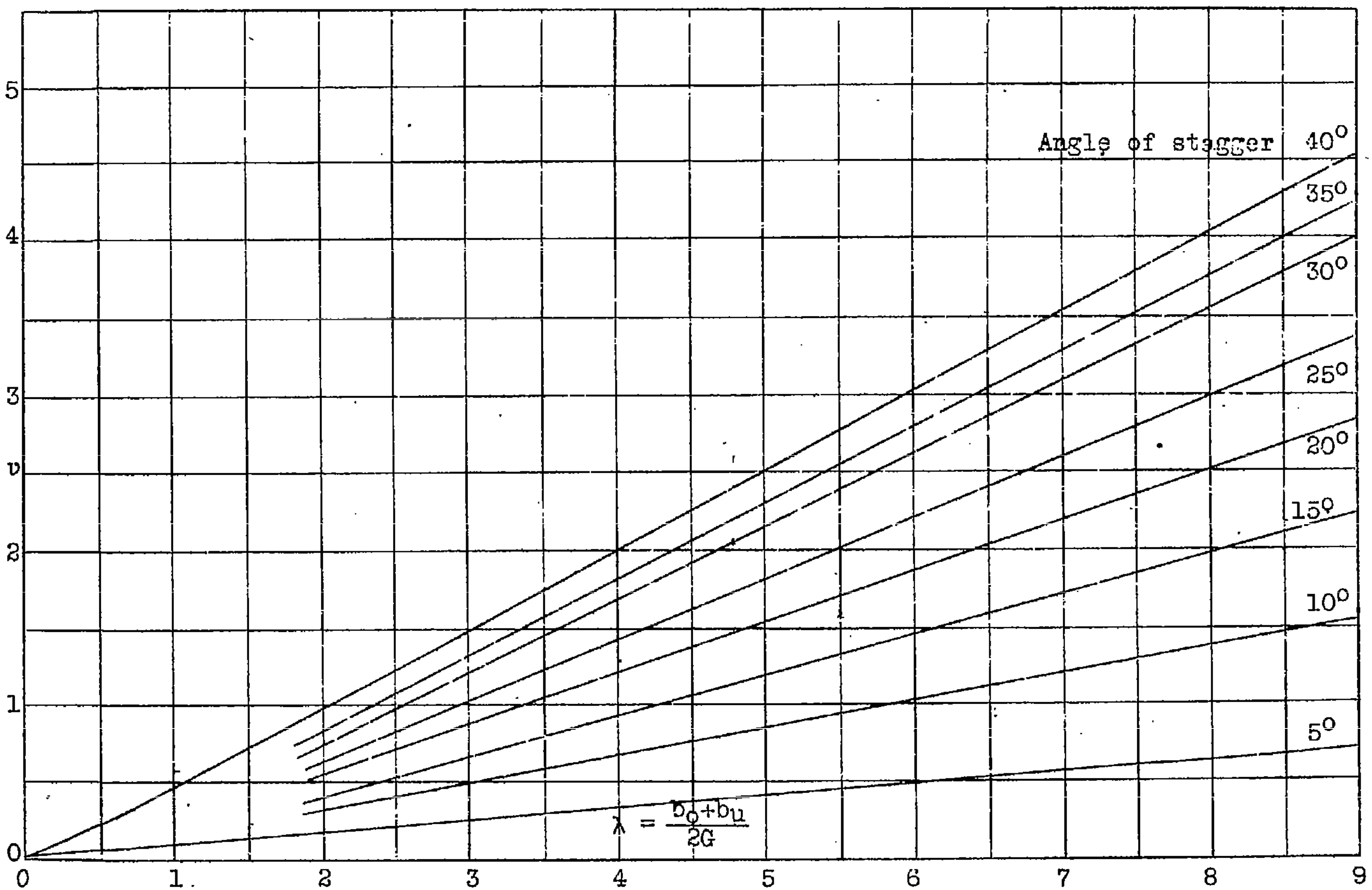


Fig.2 Variation of υ for a biplane from "Aerodynamik" by Fuchs and Hopf.

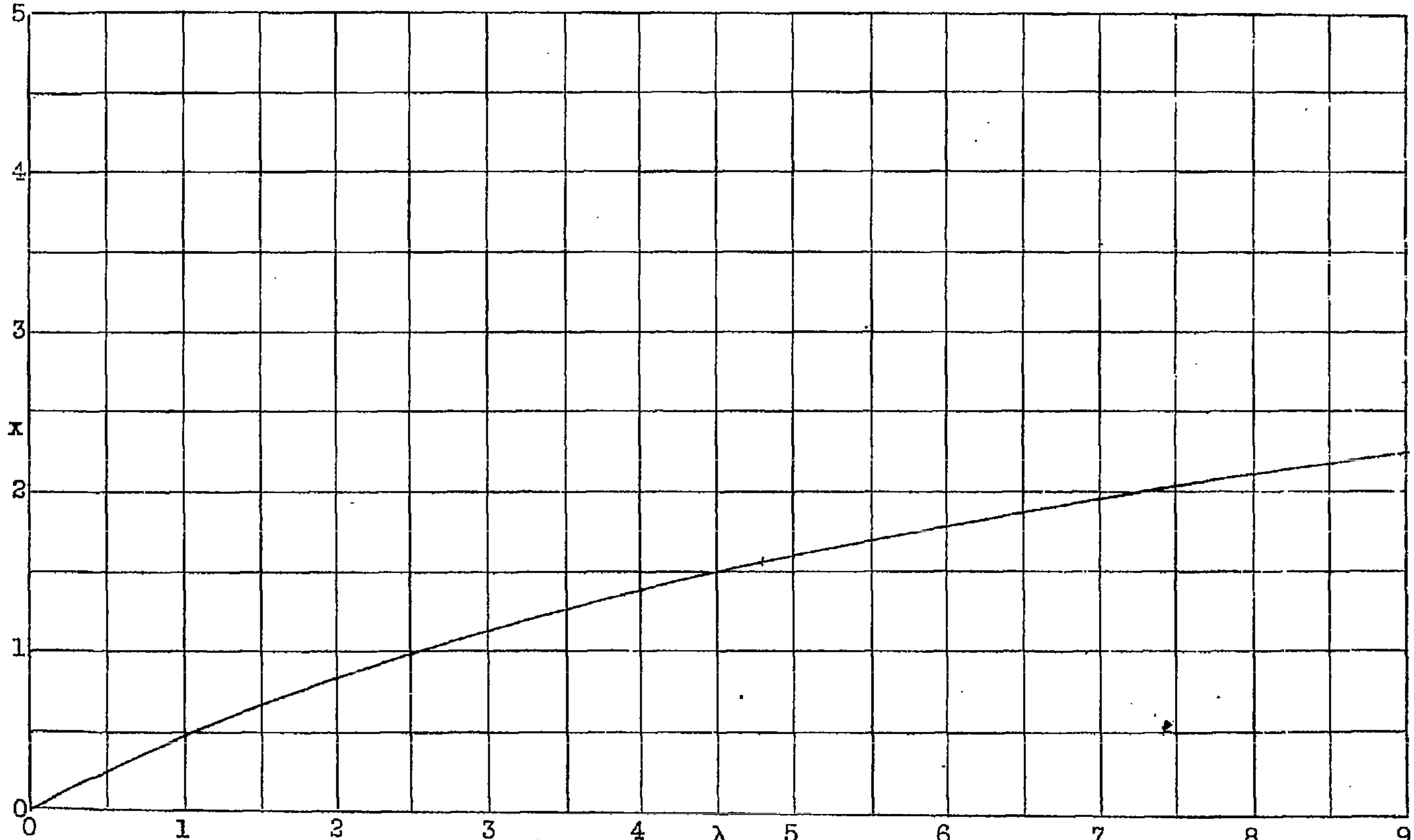


Fig.3 Variation of x for a biplane from "Aerodynamik" by Fuchs and Hopf.

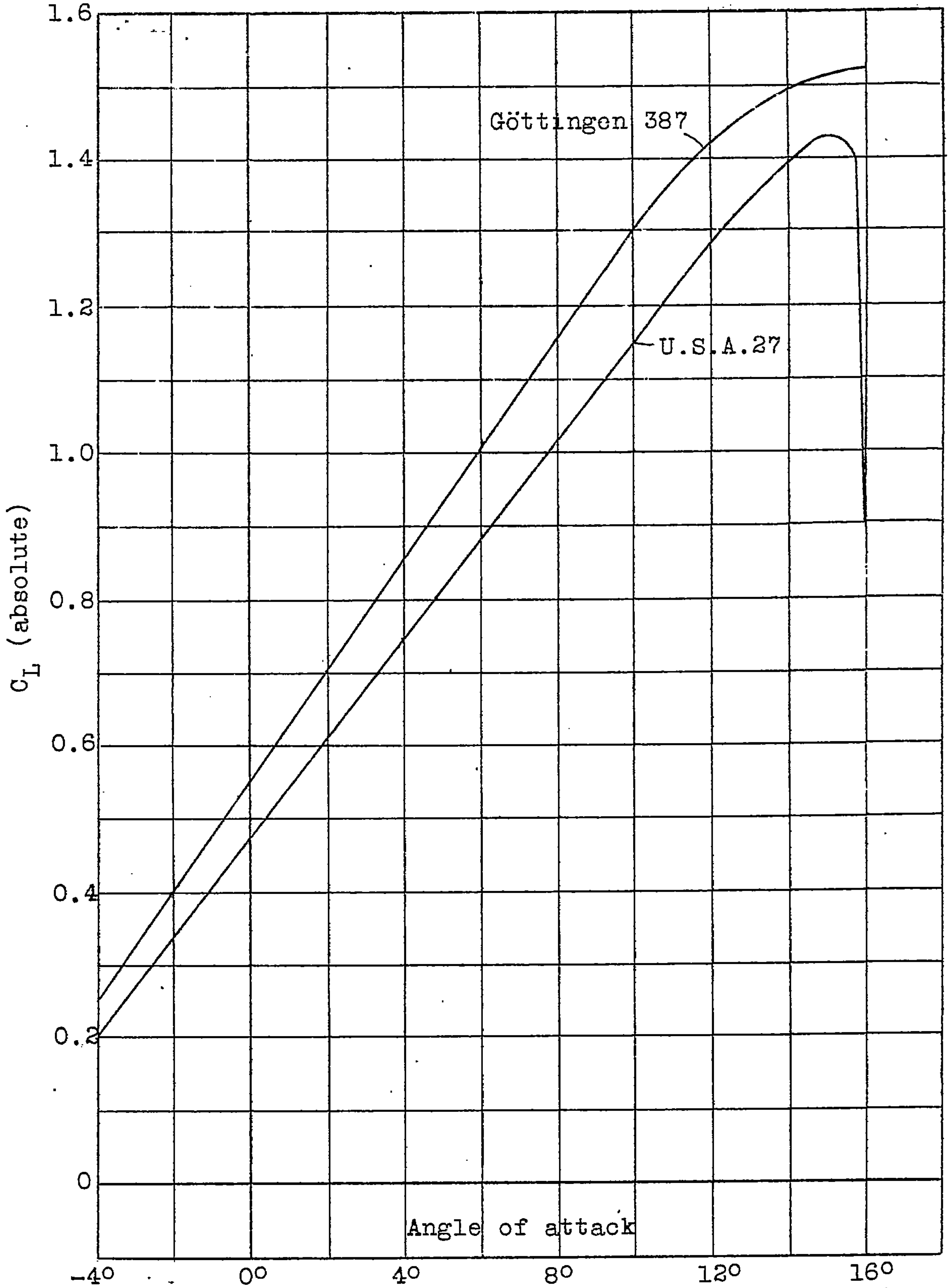
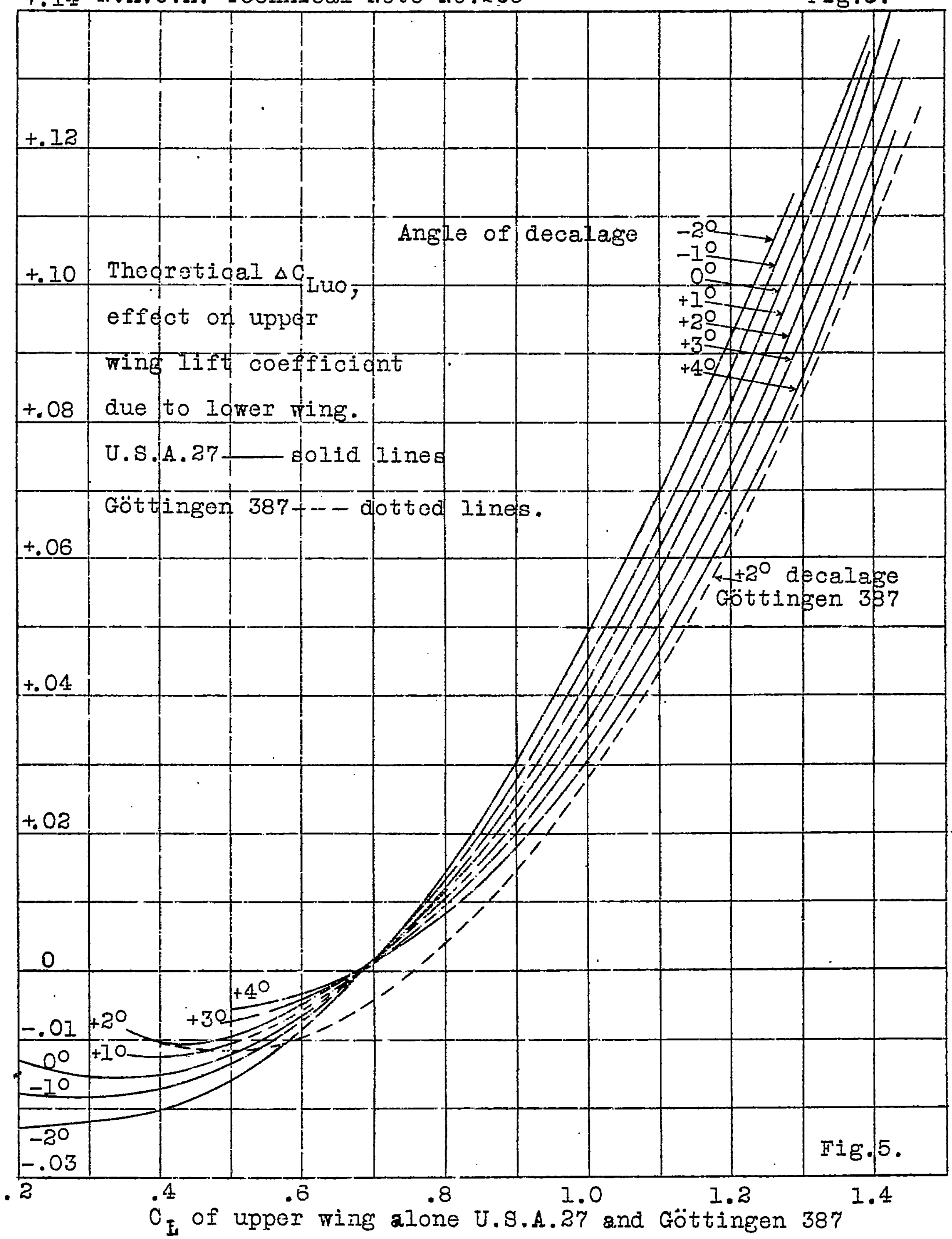
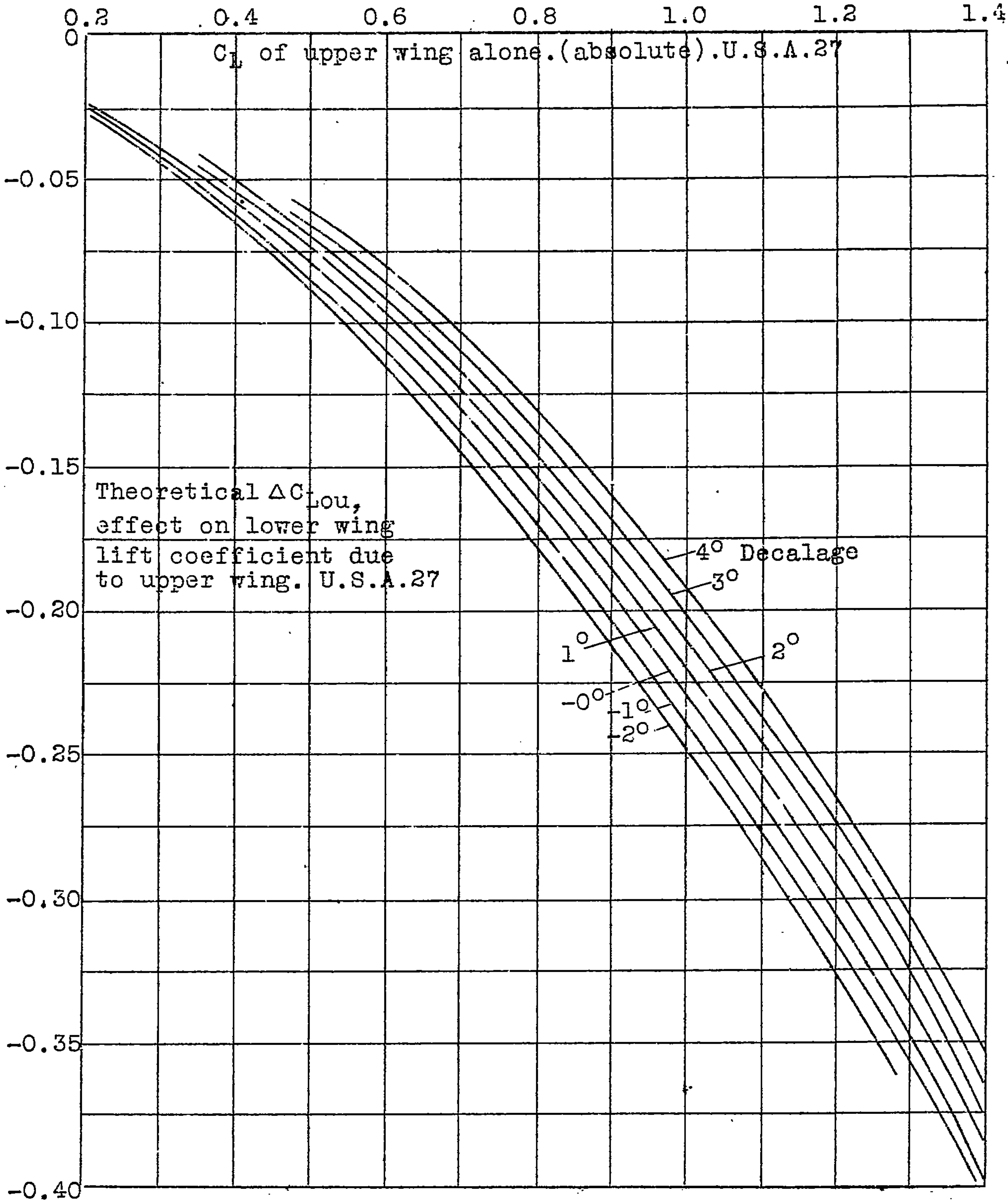


Fig.4 Göttingen 387 and U.S.A.27 airfoils. Lift characteristics. 18"×3" 40 M.P.H. N.Y.U. wind tunnel.



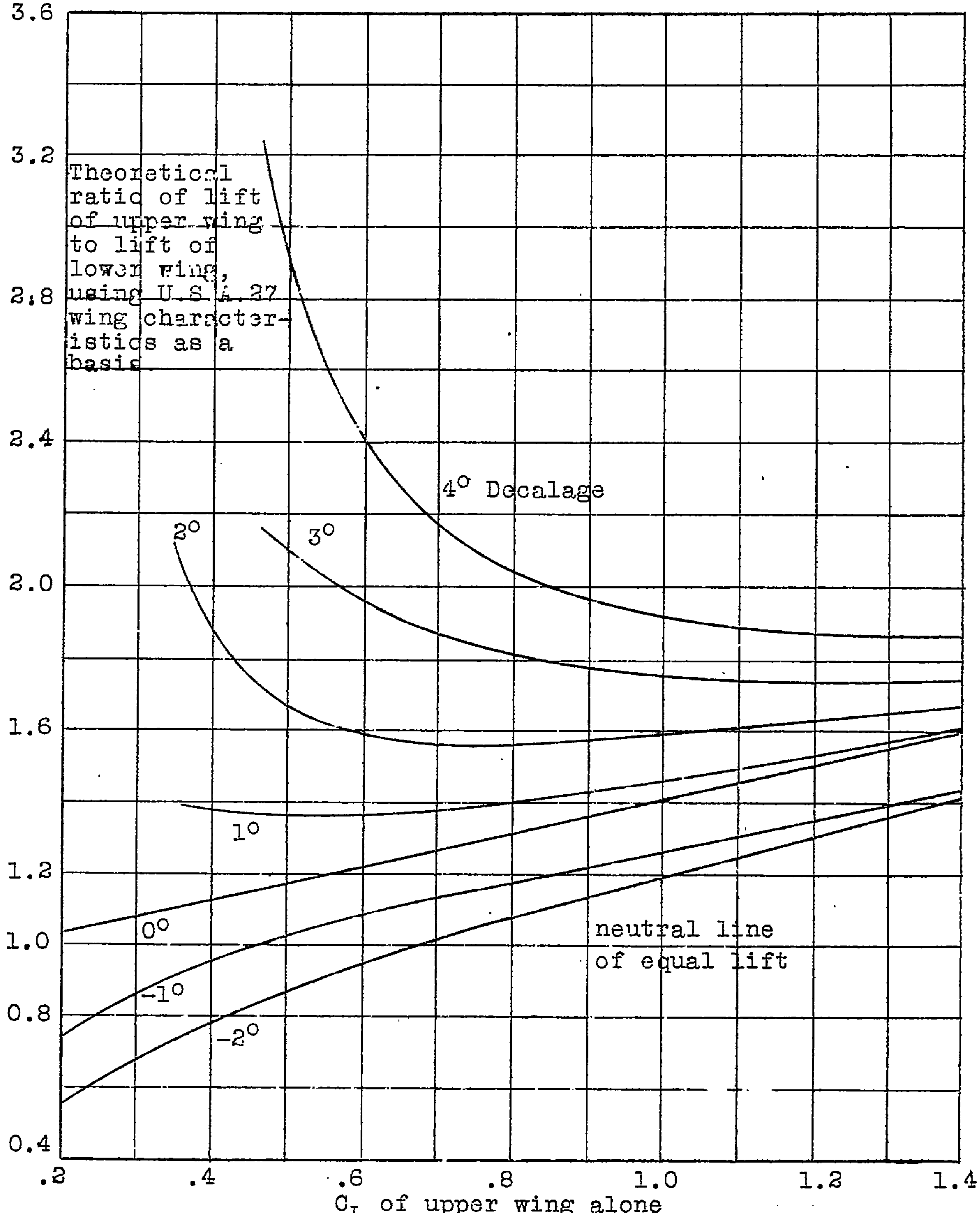
+.14
+.12
+.10
+.08
+.06
+.04
+.02
0
-.01
-.03
Angle of decalage
-2°
-1°
0°
+1°
+2°
+3°
+4°
Theoretical ΔC_{Luo},
effect on upper
wing lift coefficient
due to lower wing.
U.S.A.27 —— solid lines
Göttingen 387 - - - dotted lines.
+2° decalage
Göttingen 387
+4°
+3°
+2°
+1°
0°
-1°
-2°
.2
.4
.6
.8
1.0
1.2
1.4
C_L of upper wing alone U.S.A.27 and Göttingen 387

Fig.5.



0.2
0.4
0.6
0.8
1.0
1.2
1.4
C_L of upper wing alone.(absolute).U.S.A.27
0
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
Theoretical ΔC_{Lou},
effect on lower wing
lift coefficient due
to upper wing. U.S.A.27
4° Decalage
3°
2°
1°
-0°
-1°
-2°

Fig.6


3.6
3.2
2.8
2.4
2.0
1.6
1.2
1.0
0.8
0.4
Theoretical ratio of lift of upper wing to lift of lower wing, using U.S.A.27 wing characteristics as a basis.
4° Decalage
3°
2°
1°
0°
-1°
-2°
neutral line of equal lift
.2
.4
.6
.8
1.0
1.2
1.4
C_L of upper wing alone
Fig.7

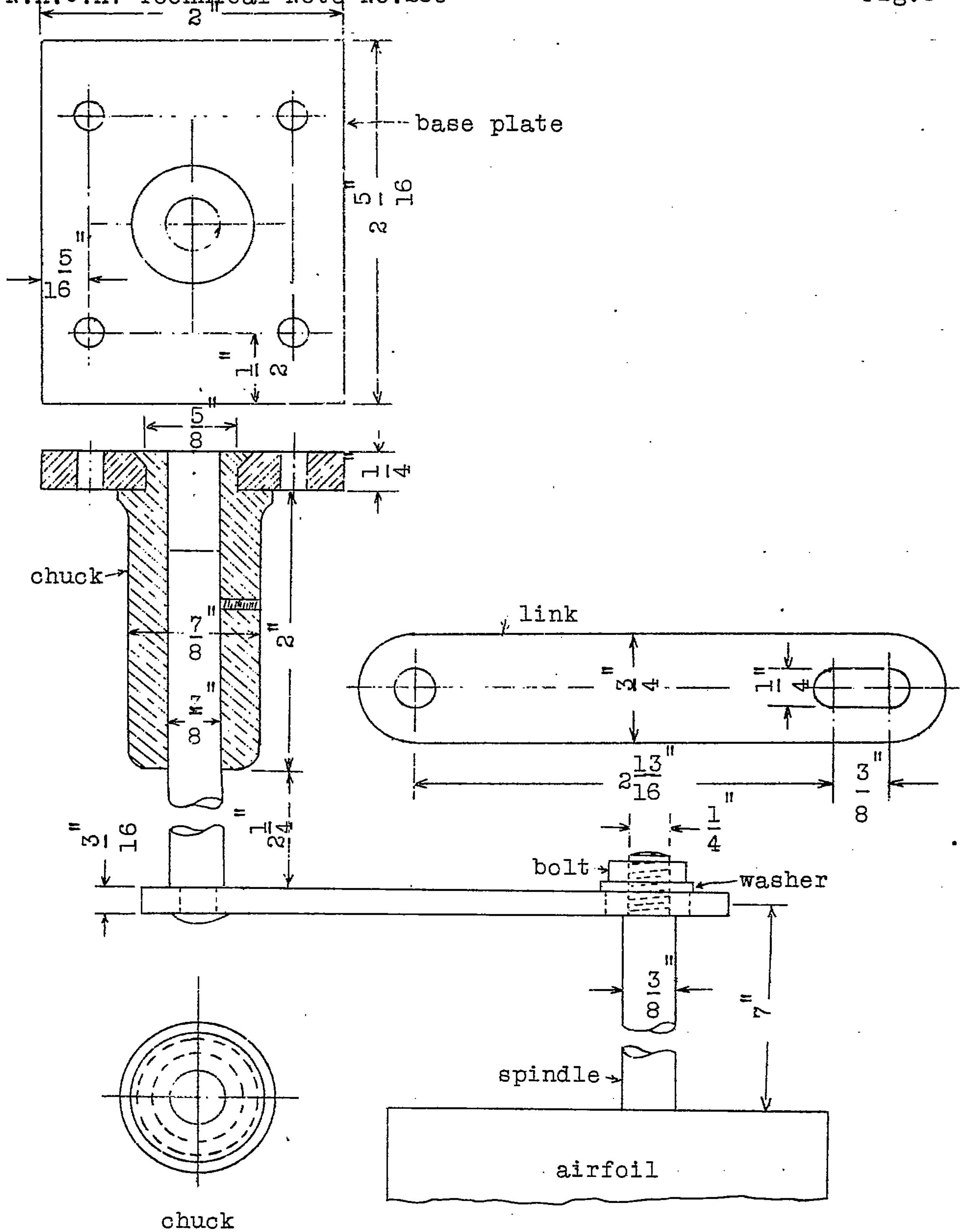
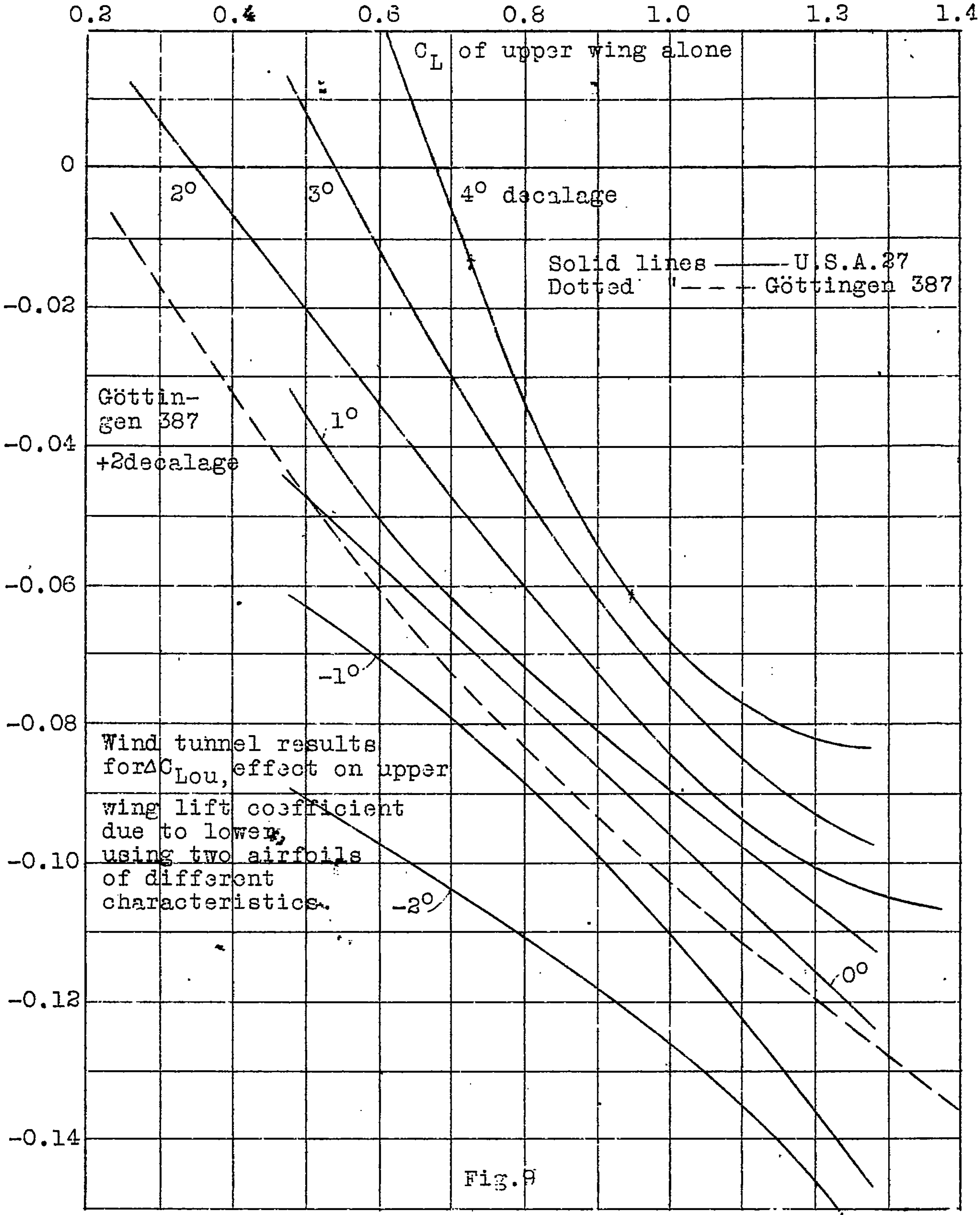


Fig.8 Mount for interfering wing in biplane test. Full scale.

Fig.8a View of wings in position to give readings



0.2
0.4
0.6
0.8
1.0
1.2
1.4
C_L of upper wing alone
0
-0.02
-0.04
-0.06
-0.08
-0.10
-0.12
-0.14
2°
3°
4° decalage
Solid lines ——— U.S.A.27
Dotted - - - - Göttingen 387
Göttingen 387
+2decalage
1°
-1°
-2°
0°
Wind tunnel results for ΔC_{Lou}, effect on upper wing lift coefficient due to lower, using two airfoils of different characteristics.

Fig.9

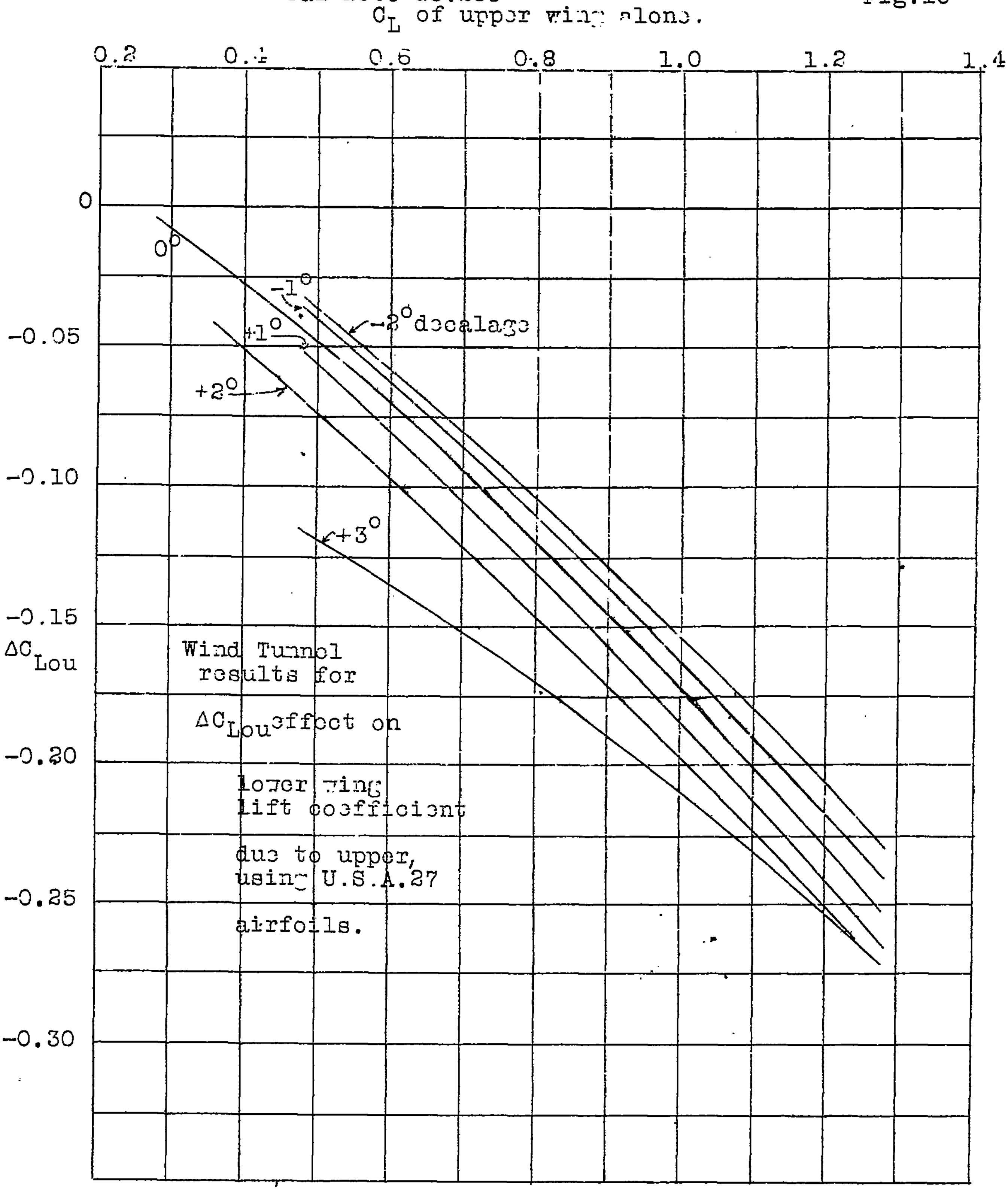

C_L of upper wing alone.
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
ΔC_{Lou}
0°
-1°
-2° decalage
+1°
+2°
+3°
Wind Tunnel results for
ΔC_{Lou} effect on
lower wing lift coefficient
due to upper, using U.S.A.27
airfoils.

Fig.10

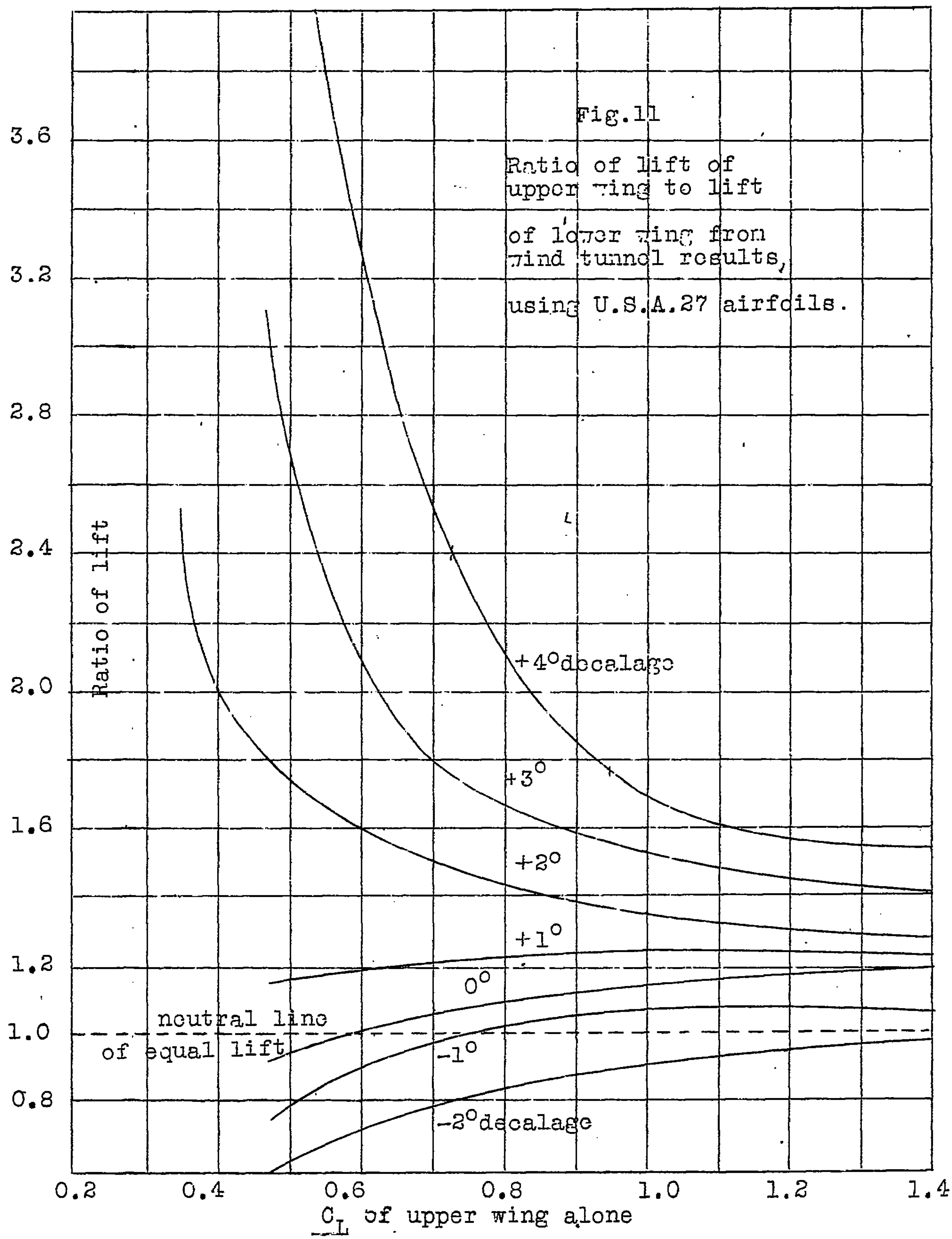
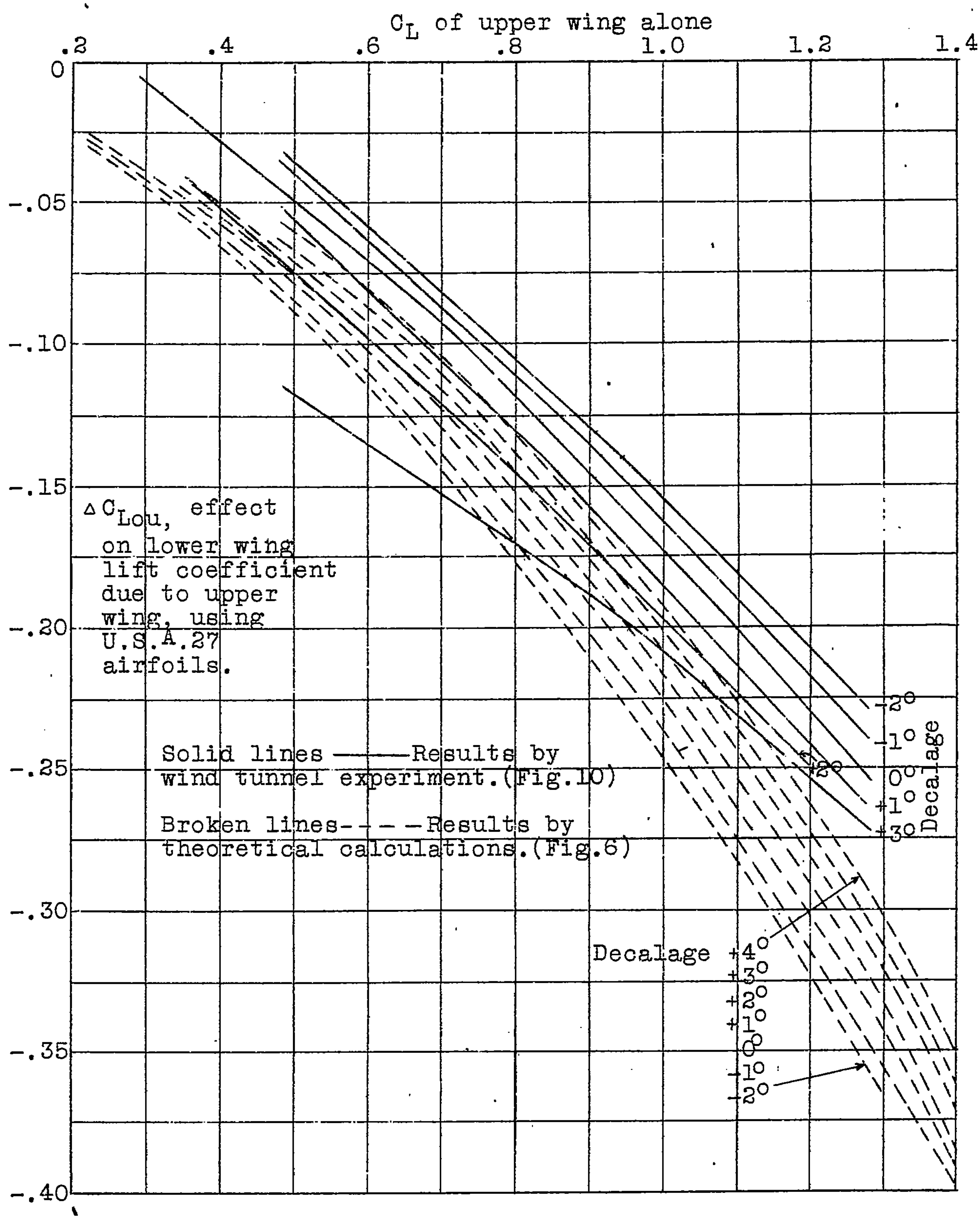


Fig.11
Ratio of lift of upper wing to lift of lower wing from wind tunnel results, using U.S.A.27 airfoils.
3.6
3.2
2.8
2.4
2.0
1.6
1.2
1.0
0.8
Ratio of lift
+4° decalage
+3°
+2°
+1°
0°
-1°
-2° decalage
neutral line of equal lift
0.2
0.4
0.6
0.8
1.0
1.2
1.4
C_L of upper wing alone



C_L of upper wing alone
.2
.4
.6
.8
1.0
1.2
1.4
0
-.05
-.10
-.15
-.20
-.25
-.30
-.35
-.40
ΔC_{Lou}, effect on lower wing lift coefficient due to upper wing, using U.S.A.27 airfoils.
Solid lines ———Results by wind tunnel experiment.(Fig.10)
Broken lines- - - -Results by theoretical calculations.(Fig.6)
-2°
-1°
0°
+1°
+3°
+2°
Decalage
Decalage +4°
+3°
+2°
+1°
0°
-1°
-2°

Fig.12.

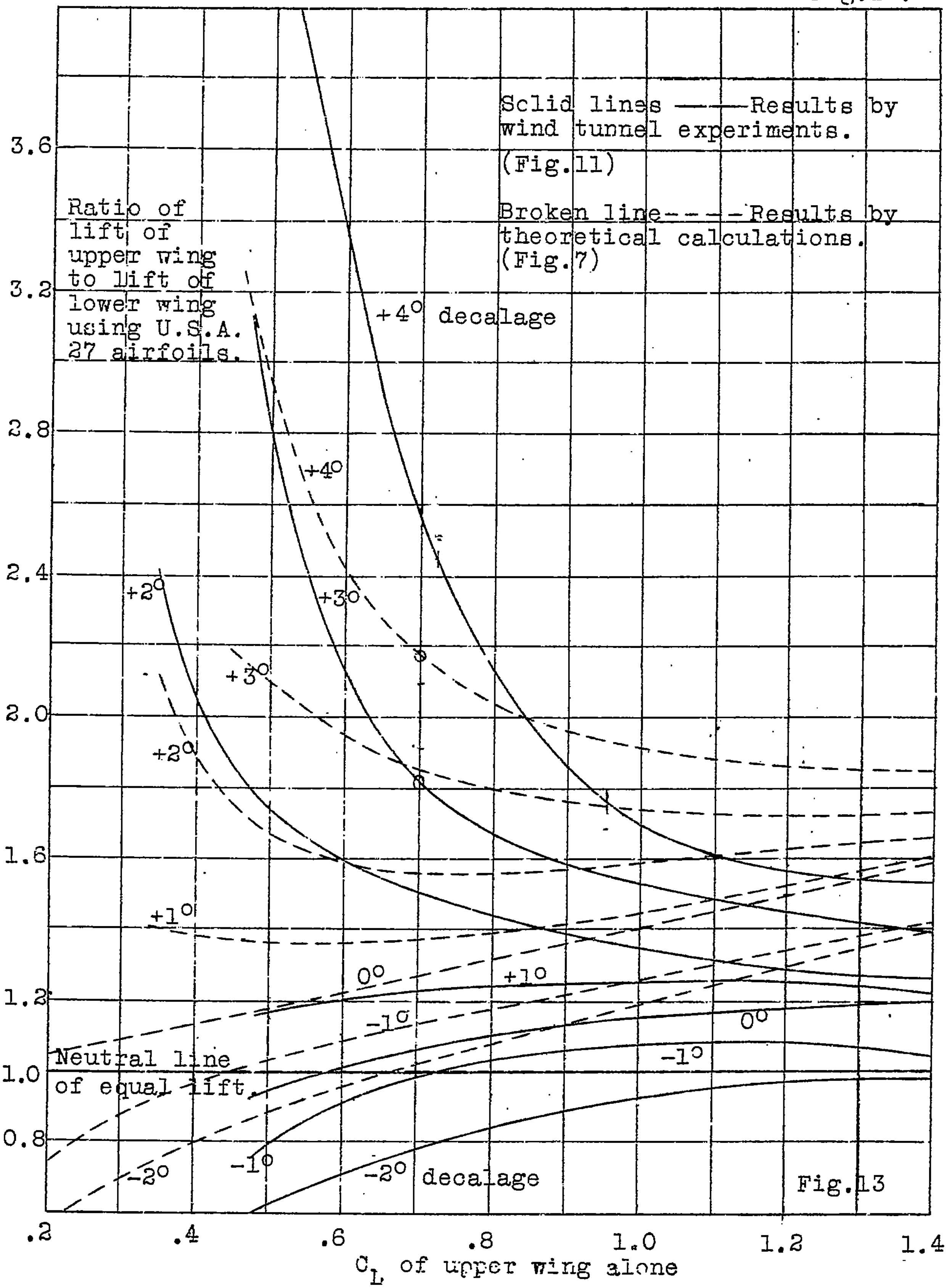

Sclid lines ——Results by wind tunnel experiments. (Fig.11)
Broken line----Results by theoretical calculations. (Fig.7)
Ratio of lift of upper wing to lift of lower wing using U.S.A. 27 airfoils.
+4° decalage
+4°
+3°
+2°
+3°
+2°
+1°
0°
+1°
-1°
0°
-1°
Neutral line of equal lift.
-2°
-1°
-2° decalage
3.6
3.2
2.8
2.4
2.0
1.6
1.2
1.0
0.8
.2
.4
.6
.8
1.0
1.2
1.4
C_L of upper wing alone

Fig.13